



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

ALIGNMENT CHARTS.

The subject of nomographic or alignment charts has received considerable attention during the last few years. Although its value as a time and labor saving device is so evident, still it has taken almost a generation since its inception by the French engineer and mathematician, M. D'Ocagne, before the American engineer has sought to profit by it. The world war brought our ordnance engineers in contact with the French engineers, and the former have learned how the latter apply the principles underlying the alignment chart to the graphical solution of some of their problems in ballistics and allied subjects. To-day, some of our manufacturers are becoming interested in these charts, and the "Department of Industrial Cooperation and Research" at the Massachusetts Institute of Technology, which is in close contact with over two hundred of these firms, has received many requests for alignment chart solutions of various simple problems which have arisen in their shop work. These solutions, because of their simplicity, can be used by the workmen in the shop with considerable facility and little chance of error.

The principles upon which the alignment chart is based are of the simplest—the idea of the representation of a function of a variable by a scale, and the well-known theorem that in similar triangles corresponding sides are proportional—ideas which the average secondary school student easily grasps. A few simple examples will suffice to illustrate some of the methods employed.*

The student is familiar with various scales—the straight thermometer scale with its uniform divisions and the uniform

* For a complete exposition of all the methods, the reader may be referred to the admirable treatise by M. D'Ocagne, "*Traité de Nomographie*," Paris, Gauthier-Villars. In the English language reference may be made to S. Brodetsky, "*A First Course in Nomography*," London, G. Bell & Sons; J. B. Peddle, "*The Construction of Graphical Charts*," New York, McGraw-Hill; J. Lipka, "*Graphical and Mechanical Computation*," Part I, "*Alignment Charts*," New York, John Wiley & Sons.

circular scale of the protractor, the non-uniform logarithmic scale of the slide rule, the non-uniform curved scale of a galvanometer dial, etc.—and he also knows that a change in the unit of measurement used in laying out such a scale will merely change its length without changing its nature or properties. Thus, in ex. 1, $OA = \frac{3}{10} p$ inches, $\frac{3}{10}$ " being the unit used in laying off the scale for the variable p ,* while $OB = \frac{1}{4} v$ inches, $\frac{1}{4}$ " being the unit used in laying off the scale for the variable v . Again, in ex. 4, $OA = 10 \log x$ inches, 10" being the unit used in laying off the scale for the variable x , so that the stroke marked $x = 2$ is at a distance $10 \log 2 = 3.01$ " from O .

Ex. 1. A manufacturer asked for a graphical solution of the following problem: *Out of a piece of cloth 36" long and of width varying from 15" to 40", rectangular pieces of cloth were to be cut varying in dimensions from 3" $\times$ 4" to 10" $\times$ 20". To find the percentage any such rectangular piece is of the whole piece.*

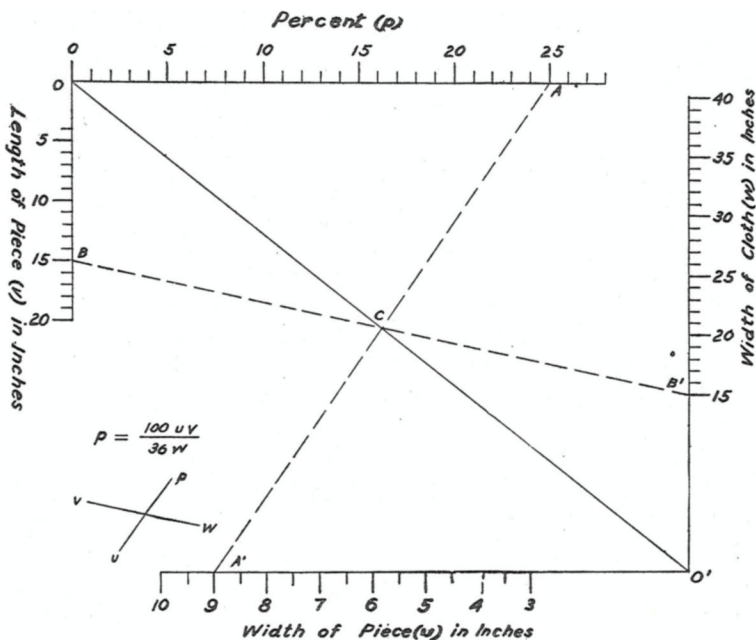


FIG. 1.

* The units are used in the construction of the original charts; the figures in the text are reductions of the original drawings.

If we let w represent the varying width of the whole piece of cloth, u and v the varying dimensions of the rectangular piece, and p the required percentage, we have

$$(1) \quad p = \frac{100uv}{36w},$$

or

$$(2) \quad p : \frac{25}{9}u = v : w.$$

In Fig. 1, we have drawn from any two convenient points O and O' , the parallel lines OA and $O'A'$ and the parallel lines OB and $O'B'$. If the dotted lines AA' and BB' are any two lines cutting the oblique line OO' in the same point C , then by similar triangles we have

$$(3) \quad OA : O'A' = OB : O'B'.$$

Now, on the two sets of parallel lines we construct scales for the variables, p , u , v , and w with m_1 , m_2 , m_3 , and m_4 inches, respectively, as units of measurements, *i.e.*, so that

$$OA = m_1p, O'A' = m_2u, OB = m_3v, O'B' = m_4w.$$

Substituting these values in (3), we note that in order that this equation should reduce to (2), we must have

$$(4) \quad m_1 : \frac{9}{25}m_2 = m_3 : m_4.$$

This is the only relation to be satisfied by our units. We choose our units so that the scales should have convenient lengths. In constructing Fig. 1, we used

$$(5) \quad m_1 = \frac{3}{10}'' , m_2 = \frac{5}{8}'' , m_3 = \frac{1}{4}'' , m_4 = \frac{1}{4}''.$$

The scales may be laid off with a properly graduated straight edge.

The chart is now ready for use. Any two dotted lines (called index lines), cutting the oblique line OO' in the same point, will cut the scales in values of p , u , v , and w satisfying the given equation. Given any three of the variables the fourth may be found. In practice the index lines are not actually drawn. The figure is drawn in ink on heavy bristol board; a blue print may be made. If, for example, u , v and w are given,

we lay a straight edge through the proper values of u and v , marking its point of intersection C with OO' , then rotate the straight edge about C until it passes through the proper value of w , and then read the required value of p from its intersection with the p -scale.

Ex. 2. A graphical solution of the following problem was called for. *Given two numbers, to find the percentage which one of the numbers is of their sum.*

If the numbers are x and y , and z is the percentage which x is of $x + y$, we have

$$(6) \quad z = \frac{100x}{x+y}, \quad \text{or} \quad x+y = \frac{x}{.01z}.$$

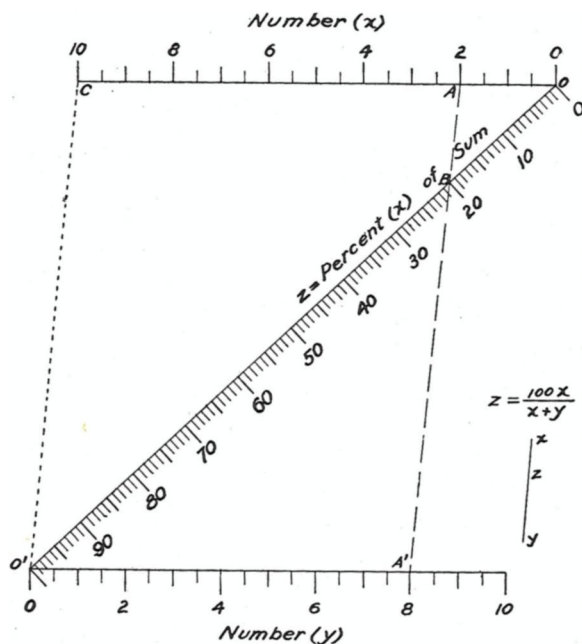


FIG. 2.

In Fig. 2, we have drawn the parallel lines OA and $O'A'$ from any two convenient points O and O' , and joined OO' . If the dotted line is any transversal cutting the parallels in A and A' and the oblique line OO' in B , and we draw the construction line $O'C$ parallel to AA' , then by similar triangles, we have

(7) $OC:OO'=OA:OB$, or $OA + O'A':OO'=OA:OB$.
Now, if on the parallel lines we construct scales for x and y with the same unit m_1 inches, so that

$$OA = m_1x, O'A' = m_1y,$$

and on OO' , starting at O , we construct a scale for z with a unit m_2 inches, so that $OB = m_2z$, then (7) becomes

$$m_1(x + y) : OO' = m_1x : m_2z,$$

and in order that this should reduce to (6), we must have

$$(8) \quad m_1 : OO' = m_1 : 100m_2$$

This is the only relation to be satisfied by the units. For the con-

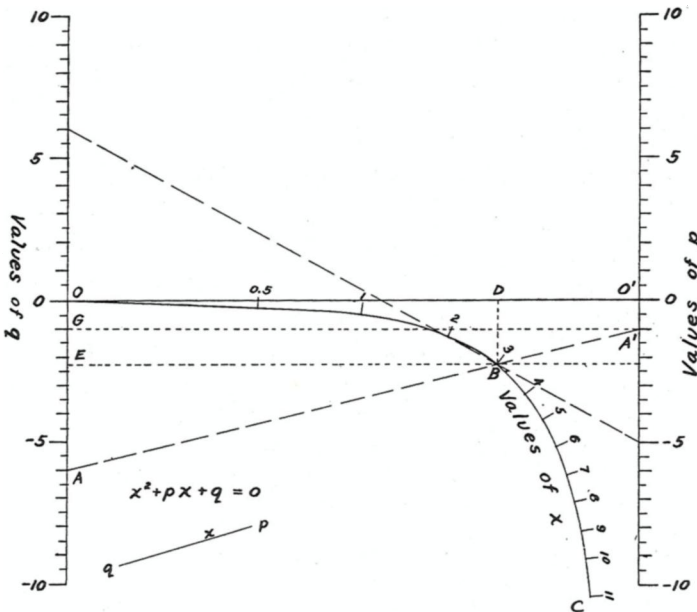


FIG. 3.

struction of Fig. 2, we choose $OO' = 10''$, $m_1 = 1''$, $m_2 = 0.1''$, so that

$$(9) \quad OA = x, O'A' = y, OB = 0.1z.$$

Any index lines will cut the three scales in values satisfying

equation (6). We have constructed the chart for values of x and y between 0 and 10.

Ex. 3. To find graphically the roots of any quadratic equation. If we consider the coefficients p and q as variables, the equation

$$(10) \quad x^2 + px + q = 0$$

may be said to represent all quadratic equations.

In Fig. 3, the directions of the lines are important—the convention being the same as in graphical algebra. We have drawn two parallel lines OA and $O'A'$ any convenient distance apart, a curve OBC , and any transversal AA' cutting the parallels in A and A' and the curve in B . The line OO' , perpendicular to the parallels, is 20" long. The lines OA and $O'A'$ carry scales for q and p respectively with units of 1", so that $OA = q$, $O'A' = p$, and p and q vary from -10 to $+10$. The curve OBC and its accompanying scale are constructed by assigning a value to x , and laying off

$$(11) \quad OD = \frac{20x}{x+1}, \quad DB = -\frac{x^2}{x+1}$$

and marking the point reached with this value of x . This process is continued for values of x from 0 to 11 for as small intervals as we please, the points being marked with their proper values of x , and a smooth curve is drawn through the points obtained.

If we draw the construction lines BE and $A'G$ parallel to $O'O$, then by similar triangles,

$$EA : GA = EB : GA',$$

$$\text{or} \quad OA - DB : OA - O'A' = OD : OO',$$

or

$$q + \frac{x^2}{x+1} : q - p = \frac{20x}{x+1} : 20,$$

or

$$x^2 + px + q = 0,$$

so that any index line will cut the scales in values of p , q , and x satisfying this equation, or any index line cutting the parallel

scales in p and q will cut the curved scale in values of x which are the roots of the quadratic equation.

We have only constructed the curve for positive values of x . If the roots are negative, we merely change the sign of p , and proceed to find the positive roots of the new equation; these will be the negative roots of the given equation. If the index line does not cut the curve the roots are imaginary, and if it is tangent to the curve the roots are equal. If the values of p and q lie beyond the limits -10 and $+10$, it is merely necessary to make the transformation $x = ax'$, and get

$$x'^2 + \frac{p}{a}x' + \frac{q}{a^2} = 0,$$

and then choose a so that the new coefficients

$$p' = \frac{p}{a}$$

and

$$q' = \frac{q}{a^2}$$

lie between -10 and $+10$.

The solution of any cubic equation $x^3 + px + q = 0$ may be solved graphically in a similar manner. The corresponding curved scale is obtained by laying off

$$OD = \frac{20x}{x+1}, \quad DB = -\frac{x^3}{x+1}.$$

Ex. 4. To build a graphical chart for multiplication and division. Let $z = xy$. This may be written

$$(12) \quad \log z = \log x + \log y.$$

In Fig. 4, we have drawn three parallel equidistant lines and any transversal cutting these in A , A' , and A'' . Then evidently

$$O''A'' = \frac{1}{2}(OA + O'A').$$

If we construct scales on these lines so that

$$OA = 10 \log x, \quad O'A' = 10 \log y, \quad O''A'' = 5 \log z,$$

where we have used $10''$, $10''$, and $5''$ as our units, and mark the scales with corresponding values of x , y , and z , then any

straight edge laid across the scales will cut out values of x , y , and z such that $\log z = \log x + \log y$ or $z = xy$. Multiplication and division are performed by the same process. The logarithmic scales are approximate reproductions of the scales

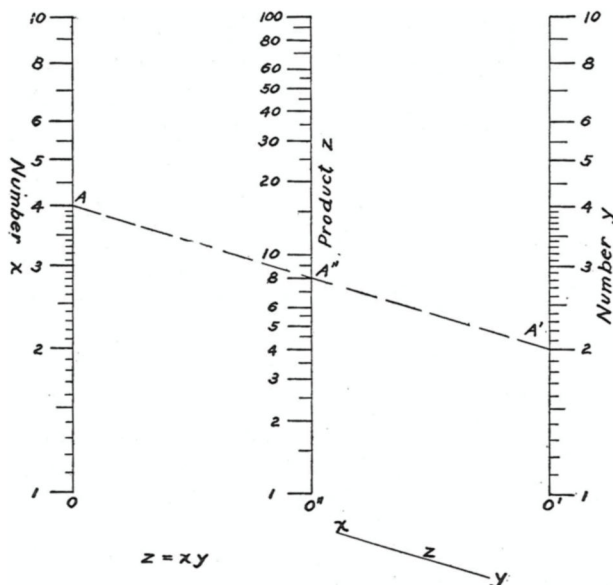


FIG. 4.

of the slide rule—the scales of the slide rule have 25 cm. and 12.5 cm. as units. Of course, as in the slide rule, the decimal point may be disregarded and its final position determined from the problem.

The method outlined in this example has a wider application than any of the other methods, for it may easily be extended to any equation in three variables of the form $z^n = ax^my^k$, where a , k , m , and n are any constants whatsoever; these constants merely determine the relative positions of the scales and the units of measurements to be used in the construction of the scales. The method may also be extended to formulas involving more than three variables.

JOSEPH LIPKA.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
CAMBRIDGE, MASS.